**APPLICATON OF AN ORIENTED NETWORK IN DETERMINING THE BEST REPLACEMENT POLICY FOR JKUAT BUSES**

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**2019**

**DECLARATION:**

This project is our original work and has not been presented for a degree in any other university.

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**DEDICATION**

To the God Almighty, Creator of heaven, earth and all that exists, and who goes beyond any comprehension. Whatever we are or will ever be we owe it to God.

**ACKWNOLEDGMENTS**

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**SYMBOLS AND ABBREVIATIONS**

Value of a bus at the beginning of year n

Average maintenance cost.

Cumulative maintenance cost.

Net Operating cost

net operating cost for year n

net operating cost for 2520 years

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**ABSTRACT**

Oriented networks has quite number of applications in the real world. For instance oriented networks could be modelled in the transport industry to find the shortest route between cities where the vertices represent the cities and the edges represent the roads connecting the cities. The current study considers the shortest path problem in oriented networks to determine the cheapest way to replace buses that have been used after a given time period at Jomo Kenyatta University of Agriculture and Technology. In the research the operating costs are modelled on an oriented network where the nodes represent the year after the purchase of the bus and the arcs represent the net operating cost. From this, the shortest path is identified in order to determine the time period for which a bus should be maintained. The bus replacement policy that minimizes the total operating cost is then deduced.

**CHAPTER ONE**

**INTRODUCTION**

* 1. **Background of the Study**

**1.1.1 Networks**

A network is a set of objects called nodes or vertices that are connected together. The connections between the nodes are called edges or arcs.

A node refers to a point in network or diagram at which lines or pathways intersect in a branch.

A directed network is a set of objects (called vertices or nodes) that are connected together, where all the edges are directed from one vertex to another. An undirected network is a set of objects (called vertices or nodes) that are connected together, where all the edges are bidirectional.

Oriented network; defined as a set of points or nodes which are connected by lines or links. A way of going from one node (the origin) to another (the destination) is called a route or path.

Example of a network; interconnection of cities

A walk consists of a finite sequence of edges of the form ,, ……,. The number m of edges is called the length of the walk. A path is a walk without any repetition of nodes. A trail is defined as a walk with no repeated edges.

Applications of network

**1.1.2 Shortest Path Problem**

Shortest path problem involves a general network structure in which the only relevant parameter is cost. The goal is to find the shortest path (the path with minimum total distance) from the origin to the destination. Finding the shortest path from an origin to all other nodes is not any more difficult than determining the shortest path from an origin to a single destination.

To determine the shortest path, undirected weighted networks are used, where the weights represents the number assigned to each node. It should be noted that the weights should all be positive

The problem of shortest paths in an oriented network has several applications. For instance, transportation or communications, such as finding the direct packets to a destination across a network. In a telecommunication problem where a message must be sent between two nodes in the quickest way possible. In social networks to find the degrees of separation between people. In chemistry and physics, oriented networks can be used to study the structure of molecules. A network makes a natural model for molecules, where vertices atoms and edge bond. Other applications include DNA sequencing, solving certain types of differential equations and approximating functions.

**1.1.3 Equipment Replacement Policy**

In an equipment replacement problem, the input is the total cash inflow/outflow for purchasing the equipment using the equipment for a certain period of time, and finally selling the equipment. This data can be transformed into a network structure by assuming that the nodes represent the timeline and the arcs represent an equipment replacement decision. The cost of an arc between two nodes separated by k years is the total cost of buying, keeping, and then selling the equipment for k years. It also represents the total net operating cost. The goal in an equipment replacement problem is to determine the best equipment replacement policy over a t-year planning horizon, which is equivalent to solving a shortest path problem.

**1.1.4 Shortest-Path Algorithm**

Let nodes Y1…..Y5, represent the number of years and the number of years arcs represent the operation cost at the beginning of each year. The cost associated with each branch is the net operating cost. The shortest path algorithm below is used to determine the minimum operation cost.

STEP 1: Construct a master list by tabulating under each node in ascending order of cost, the branches incident on it. Each branch under a given node is written with that node as its first node. Omit from the list any branch having the source as its second node or having the sink as its first node.

STEP 2: Star the source and assign it the value zero (0). Locate the cheapest branch incident on the source and circle it. Star the second node of this branch and assign this node a value equal to the cost of the branch. Delete from the master list all other branches that have newly starred node as second node.

STEP 3: If the newly starred node is the sink, go to Step 5 if not go to Step 4.

STEP 4: Consider all starred nodes having uncircled branches under them in the current master list. For each every node, add the value assigned to the node to the cost of the cheapest uncircled branch under it. Denote the smallest of theses sums as M, and circle that branch whose cost contributed to M. Star the second node of this branch and assign it the value M. Delete from the master list all other branches having this newly starred node as second node. Go to Step3.

STEP 5: Z\* is the value assigned to the sink. A minimum cost path is obtained recursively, beginning with the sink by including in the path each circled branch whose second node belongs to the node.

The best replacement policy is then deduced.

**1.2 Statement of the Problem**

The current study applies the shortest-path algorithm on an oriented network in determination of the cheapest replacement policy for JKUAT buses.

**1.3 Justification**

When operational efficiency of an item deteriorates with time, it is economical to replace the same with a new one. For example, the maintenance cost of a machine increases with time and a stage is reached when it may not be economical to allow the machine to continue in the system.

A vehicle tends to wear out with time due to constant use. More money needs to be spent on it on account of increased repair and operating cost. A stage comes when it becomes uneconomical to maintain the vehicle and it is better to replace it with a new one. Here the replacement decision may be taken to balance the increasing maintenance cost with the decreasing money value to the vehicle, with the passing of time. The JKUAT transport department operates a number of buses used for transportation. With time, these buses become more expensive to maintain because of deterioration due to wearing out of major parts of the bus. They are then either scrapped off or their spare parts used in repair of other vehicles. This research here aims at finding the appropriate time of replacing the buses after a given time of maintenance.

**1.4 Objectives**

**1.4.1 General Objective**

To determine the best replacement policy for JKUAT buses using oriented networks.

**1.4.2 Specific Objectives**

* To model the problem as a shortest-route problem on an oriented network.
* To determine a bus replacement policy that minimizes the total operating cost for JKUAT.

**CHAPTER TWO**

**LITERATURE REVIEW**

This chapter reviews some previous studies related to the current study.

Aima (2000) compared two machines A and B in terms of their costs and operating costs. The cost of every machine after every first year and subsequent years was estimated, where the operating cost for machine A was higher than that of B. Machine A became worn out thinking of replacing it with B assuming that both machines had no resale value and future costs were not discounted.

Broadal et.al (2004) used a time-dependent approach which constructed the time-dependent digraph in which every node represented a station and two nodes connected by an edge if the corresponding stations are connected by an elementary connection. The cost of edges assigned “on the fly” the cost of an edge depended on the time in which the particular edge will be used by the shortest path algorithm.

Sven Peyer et. al (2007) generalised Dijkistra’s algorithm for finding shortest path with application VLSI routing. They proposed that for undirected graphs, a linear running time can be achieved for integral lengths or on average in a randomised setting. Their new algorithm provided a speed up technique for Dijkistra’s algorithm in two ways. First it can directly be applied to propagate distance labels through a graph. Second application of generalized Dijkistra’s algorithm is the goal oriented search.

Disser et. al (2008), studied multi criteria shortest paths in time-dependent train networks their research was focused on the German train company. They used the shortest path algorithm to determine the time information of the German Company in order to speed up train schedules to realise their objective.

Gage (2013) investigated the equipment maintenance and replacement decision making process. In an industry or organisation for an instance transport department decisions have to be made when to purchase new equipment and when to stop maintaining the existing ones. Gage used the shortest path algorithm to determine when to replace or continue maintaining the given equipment.

Laifeng et. al (2014) researched on basis of network model. They investigated equipment from the time of construction to the time of changing the equipment. They represented on a network the operating cost and the time from purchase date. They used the shortest path algorithm in order to find out the most suitable time equipment should be replaced.

Omar et. al (2015) researched on efficient routing methods allowing travellers to reach destinations through intricate multimodal transportation scheme. They compared this with the shortest path algorithm in order to find out the efficient routing methods.

Abdel Fattah IDRI et.al (2017) worked on a new time-dependent shortest path algorithm for multimodal transportation network. They introduced a new time-dependent shortest path algorithm for multimodal transportation network, which took into account the concept of closeness to the target node as heuristic to drive the search towards its destination. The proposed algorithm was a goal-oriented single-source single-destination algorithm based on computing a virtual path which is basically an Euclidean distance from the source to the target aiming at a restriction of the search space*.*

Abensur et.al (2018) investigated a hybrid mathematical model in order to support long term planning. They based this on dynamic programming and the shortest path algorithm. Through dynamic programing and presentation on oriented networks they investigated the equipment replacement policy and decision making.

**CHAPTER THREE**

**RESEARCH METHODOLOGY**

**3.1 Data Collection**

A questionnaire was used to collect data from the transport department. The data collected was for two buses (KBJ 100U and KBJ 129U). The data entailed purchase price of the bus, maintenance cost for each calendar year (general expense cost). The depreciation rate for each bus was also obtained (25% per year). The data collected was from 2010-2018. The buses were bought on 17/12/2009. The purchase price was KSH.12, 300,000.00 each. During these years prices of new buses and maintenance cost have remained constant on average. The current JKUAT replacement policy entails that a bus is replaced when the total cumulative maintenance cost sums up to more than half of the purchase price.

**3.2 Analysis of Data**

The data collected is as tabulated in Table 3.2.1 below. The net operating cost is given as……

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| YEAR |  | Maintenance cost for KBJ 100U | Maintenance cost for KBJ 129U |  |  |  | |
|  | 12300000.00 | 23000.00 | 18755.63 | 20877.82 | 20877.82 | 3095877.82 |
|  | 9225000.00 | 89670.85 | 87790.65 | 88730.75 | 109608.57 | 5490858.57 |
|  | 6918750.00 | 763609.20 | 666301.49 | 714955.35 | 824563.91 | 7935501.41 |
|  | 5189062.50 | 83251.00 | 223647.07 | 153449.04 | 978012.95 | 9386216.07 |
|  | 3891796.88 | 179112.00 | 278028.81 | 228570.41 | 1206583.35 | 10587735.69 |
|  | 2918847.66 | 587737.71 | 236008.74 | 411873.23 | 1618456.58 | 11729320.83 |
|  | 2189135.74 | 596427.07 | 243924.96 | 420176.02 | 2038632.59 | 12696780.78 |
|  | 1641851.81 | 723450.50 | 434500.13 | 578975.32 | 2617607.91 | 13686219.05 |
|  | 1231388.85 | 863400.25 | 345789.30 | 604594.78 | 3222202.68 | 15522202.68 |

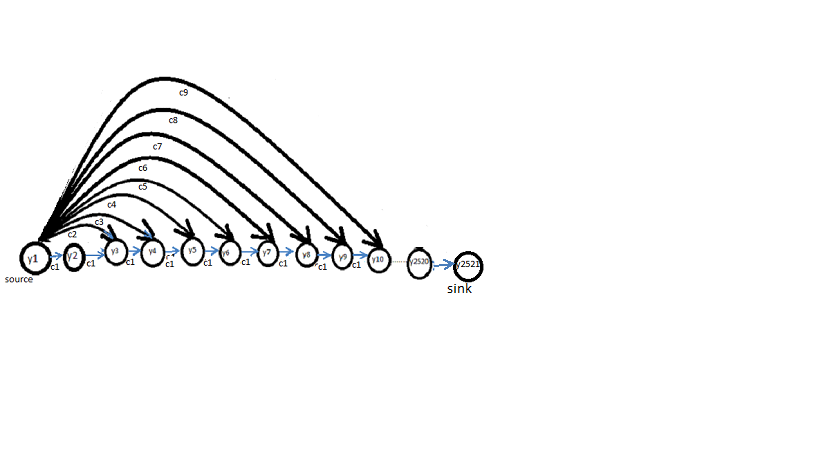
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**3.3 Modelling of Data**

The net operating cost () in Table 3.2.1 above was modelled as in the following network.

In this case the aim of is to determine all possible -year patterns (since the data available is for only nine years, ­­) after which a bus should be replaced. In order to do this it is necessary to find the least common multiple of all integers where ­­, which is 2520. So the network will have at least 2521 nodes.

**3.4 Shortest Path**

Now, the idea is to find all the distances (­­). The values of (­­) are as in Table 3.4.1 below.

|  |  |  |
| --- | --- | --- |
| No of Years Maintaining a Bus From Time of Purchase |  |  |
| 1 | 3095877.82 | 7801612093.80 |
| 2 | 5490858.57 | 6918481791.90 |
| 3 | 7935501.41 | 6665821184.40 |
| 4 | 9386216.07 | 59133116124.10 |
| 5 | 10587735.69 | 5336218789.65 |
| 6 | 11729320.83 | 4926314749.78 |
| 7 | 12696780.78 | 4570841082.01 |
| 8 | 13686219.05 | 4311159000.76 |
| 9 | 15522202.68 | 4346216750.40 |

One can notice that decreases with time up to eight years after which it increases. The reason for this is due to deterioration of the bus. The shortest path from vertex (source) to vertex (sink) in Figure 3.3.1 above is the smallest value of in Table 3.4.1.

This compares with argument (Quote of the book)

**CHAPTER FOUR**

**RESULTS, DISCUSSION AND RECOMMENDATIONS**

**4.1 Results**

The net operating cost decreases up to a certain point after which it starts increasing. It is at this point that one should consider replacing the bus. The reason for the increase in the net operating cost is as a result of deterioration of the bus, therefore would cost more to maintain.

**4.2 Discussion**

Recall that the JKUAT management policy is that of replacing a bus when the accumulative total maintenance cost becomes more than half the purchase price of the bus. However this policy is not applied since they have buses that are old and which have gone beyond the requirement and are still being used and maintained. From our study, they should consider replacing a bus when it attains eight years from the time of purchase. This is the point after which the maintenance cost suddenly starts to increase. The JKUAT transport department should consider using and maintaining a new bus for eight years after which it should be replaced.

**4.3 Recommendations for Future research**

Carry out a similar research using linear integer programming formulation. Deterministic equipment replacement problem (which is normally solved through the use of dynamic programming) as 0-1 linear programming problem is formulated.

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